Surface Smoothing Using Kriging

AutoCAD® Civil 3D® has the ability to add data points to a surface based on mathematical criteria. This gives you the ability to strengthen the surface definition in areas where data may be sparse or where you would like to generate data extending beyond the current TIN. Civil 3D accomplishes this using Natural Neighbor Interpolation (NNI), a geometric based procedure, or Kriging, a statistical method. NNI is straightforward, operates within the current TIN boundary and requires no knowledge of the surface distribution. Kriging allows you to fill in areas within the TIN or extrapolate beyond the existing TIN boundary. Using Kriging effectively requires knowledge of the surface distribution and an understanding of the statistical process involved. Although more complex, Kriging is more adaptable and can yield more accurate results.

Surface Distributions

The following table shows the basic statistics for two surface populations represented by point elevation data. You can see that they look quite similar. Even the histograms drawn below the table give the impression that the surfaces are similar. The histograms plot the number of points (Y axis) versus the elevation range (X axis).

<table>
<thead>
<tr>
<th>Population</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elevation Point Count</td>
<td>16137</td>
<td>16137</td>
</tr>
<tr>
<td>Average</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>20.2</td>
<td>20.2</td>
</tr>
<tr>
<td>Median</td>
<td>100.3</td>
<td>100.8</td>
</tr>
<tr>
<td>10 Percentile</td>
<td>73.6</td>
<td>73.8</td>
</tr>
<tr>
<td>90 Percentile</td>
<td>125.73</td>
<td>124.31</td>
</tr>
</tbody>
</table>

![Histogram A](image1.png) ![Histogram B](image2.png)
However, when we look at the surface plots, we see that they are dissimilar. The surface on the left representing population A is “rougher”, having more abrupt gradations between elevations and spatial variations between the high and low areas, while the surface on the right representing population B is more homogenous. Since both surfaces have the same standard deviation, we cannot say that one surface is more variable than the other. The difference is in the texture; the way the variations relate to each other in space. Standard statistical methods do not take into account this spatial relationship.

The Variogram

Kriging relies on a statistic called the variogram. The variogram graphically represents the spatial continuity of the surface population you are working with. The variogram plots shown below illustrate the different nature of the two surfaces. Each variogram shows the experimental data as connected dots and the theoretical model as a solid line. You can see the marked difference in the initial slopes. The variogram on the left (population A) has a steep slope and then levels off sharply, while the one on the right (population B) shows a more gradual slope and leveling. The point where the variogram levels off is called the sill. What the variogram for population A shows is that there is only a short distance where continuity exists between data points. In other words, the surface is rough or discontinuous.
The mathematics of the derivation of the variogram are complex and rather dry, so we’ll skip most of that and just look at the equation in its final form with a brief explanation of what it represents. The equation for the variogram is:

$$\gamma(\Delta x, \Delta y) = \frac{1}{2} e\left[\frac{\left(Z(x + \Delta x, y + \Delta y) - Z(x, y)\right)^2}{\Delta x \Delta y}\right]$$

This form of the equation often is called the semi-variogram, since the usual derivation ends up with the left side of the equation being written as $2\gamma()$. Regardless of the form, the result is the same.

So, what's this all mean for our surface? Well, the first thing to note is that the variogram is a function of separation of the points (the $\Delta x$ and $\Delta y$) rather than the actual location $(x, y)$ of the point. The right hand side of the equation represents the expectation ($e[ ]$ is called the expectation operator) of the inverse measure of the statistical dependency of the random variables $Z(x + \Delta x, y + \Delta y)$ and $Z(x, y)$. This expectancy gives us the theoretical variogram. There are an infinite number of models that can be derived for the theoretical variogram. In practice, only a few are used. Civil 3D 2011 makes use of five variogram models; Linear, Monomial, Spherical, Exponential and Gaussian. When you undertake surface smoothing in Civil 3D 2011, you will compare one of these theoretical models to the experimental data which is gathered from actual surface points. While the variogram equation shown above is great for the theoretical model, it isn’t much use for our experimental surface data. For that, we need another equation to calculate our experimental variogram. That equation is:

$$\gamma(\Delta x, \Delta y) = \frac{1}{2N(\Delta x, \Delta y)} \sum_{(i,j) \in (\Delta x, \Delta y)} (z_i - z_j)^2$$

The term $(\Delta x, \Delta y)$ as we mentioned before, represents the spatial difference, sometimes called the separation vector, of the points included in the set of the points gathered from the surface population. The experimental variogram is calculated for a particular separation vector by averaging one-half the difference squared of the z-values over all pairs of the observations in the set whose separation vector is approximately equal to that particular vector.

Once the experimental variogram has been calculated, it can be compared to a theoretical variogram so that we can derive the additional points we require to smooth the surface in the area of interest. One thing to note when using Kriging is that this is a very computation intensive process, since processing even a moderate number of points selected from the surface generates a significant number of iterations. In fact, if you select n points, there will be $n(n - 1)/2$ pairs processed.

Using Kriging in Civil 3D 2011

Let’s look at an example of surface smoothing using the Kriging procedure in Civil 3D 2011. Here’s a sample surface shown using a TIN style.
We'll select the **Smooth Surface** command from the surface edits right click command popup menu.

This command launches the **Smooth Surface** dialog shown below. Until **Kriging** is selected as the smoothing method, the dialog will appear slightly different.

Now, let's look at each of the areas within this dialog. Clicking the left mouse button in a field will activate any value selection methods available for that field (drop down lists, drawing object selection buttons, etc.)
Select *Krigeing* as the smoothing method.

The *Krigeing Method* section allows us to select the variogram model, point selection method and the points themselves.

**Variogram Model**  
Civil 3D contains five theoretical variogram models.

<table>
<thead>
<tr>
<th>Variogram Model</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td><img src="image1.png" alt="Graph" /></td>
</tr>
<tr>
<td>Monomial</td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
<tr>
<td>Spherical</td>
<td><img src="image3.png" alt="Graph" /></td>
</tr>
<tr>
<td>Exponential</td>
<td><img src="image4.png" alt="Graph" /></td>
</tr>
<tr>
<td>Gaussian</td>
<td><img src="image5.png" alt="Graph" /></td>
</tr>
</tbody>
</table>
Once you’ve selected the input points for the experimental variogram, Civil 3D 2011 will plot these points and the theoretical variogram in the window at the bottom of the dialog. The Linear variogram is the default model for this dialog.

**Point Selection Method**
This field is a dropdown that allows you to specify the points to use for the surface smoothing extrapolation. The choices are:
- **Select Points** which selects all points inside a rectangle, polygon, surface, or parcel.
- **Random Points** which selects random points inside a rectangle, polygon, surface, or parcel.
- **Select All Points** which selects all points on a surface.

**Select Points**
This field allows you to select the actual points in the drawing. It is available only when either the Select Points or Random Points parameter is selected. Click the button and follow the command line or dynamic cursor prompts to select the points. Once the points are selected, the number of selected points will be visible in the field.

The **Point Interpolation / Extrapolation Output** section lets us choose our output parameters.

**Output Locations**
This field specifies the output location for the points.
- **Grid Based** outputs points on a grid defined within specified polygonal areas selected in the drawing.
- **Centroids** outputs points at the existing surface triangle centroids within specified polygonal areas selected in the drawing.
- **Random Points** outputs a specified number of random points within polygonal areas selected in the drawing.

**Select Output Region**
This field specifies the area where the points will be output. Click the button and follow the command line or dynamic cursor prompts to select the area.

**Grid X - Spacing**
This field specifies the X distance between the grid lines. This is available only when the Grid Based option for point output is selected. Click the button to digitize the spacing in the drawing area.

**Grid Y - Spacing**
This specifies the Y distance between the grid lines. This is available only when the Grid Based option for point output is selected. Click the button to digitize the spacing in the drawing area.

**Grid Orientation**
This field lets you specify the orientation direction for the grid in the Y and X directions. It is available only when the Grid Based option for point output has been selected. Click the button to pick two points in the drawing from which to define the orientation direction.

**Number of Output Points**
This field displays the number of points that will be output to the surface.

Once the input parameters have been chosen and the original surface points selected, Civil 3D 2011 will draw the variogram model (red line) and the experimental variogram (points). You can compare the points to various theoretical models by selecting the model from the Variogram Model field dropdown. To the left of the variogram plot are three parameters relating to the variogram. More specifically, these are:
Parameter A
This is the distance along the X axis where the sill takes effect. This is often called the *lag* of the variogram. The Linear variogram model has no sill, so if you choose this model, you will not see this field.

Parameter C
This is the distance along the Y axis where the sill takes effect. This is often called the *scale* of the variogram.

Nugget Effect
The nugget effect is measured by the distance above zero on the Y axis where the experimental variogram meets that axis. The nugget effect represents the distance between points where there is no expectation of measurable variance. As the name suggests, it represents a nugget or concentration of data. Originally, Kriging was developed for use in the mining industry (Krige, for whom this analysis is named, is a South African mining engineer), to try to estimate the size and grade of an orebody based on data from a number of drill holes. One of the considerations was, for any ore intersection in a drill hole, did this intersection represent a larger volume of ore or was this just a coincidental intersection with a small concentration of ore. The nugget effect represents this lower limit of confidence in the separation vector between pairs of points.

Here’s a smoothing done on our sample area using a Gaussian model. I’ve used the TIN triangle centroids as outputs, and have adjusted the scale and the lag of the variogram to try to fit the data better. Adjusting the scale and lag is a bit of alchemy; you’ll have to experiment to find the combination that works best for the surface sample. You also should look at different models to see how they fit the data. You can see that there are 270 output points generated by this Kriging.
Here is a before and after image of our surface TIN for the sample area.

![Original Surface](image1)

![Smoothed Surface](image2)

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Jeff, a senior civil and geospatial consultant with IMAGINiT, has over thirty years experience in the mining and engineering sectors. He has been involved with AutoCAD and the Autodesk civil software applications since their earliest days, and has consulted on the development of standards and procedures, CAD management, database and technical applications, training and implementation, and project management. Jeff is recognized for his work in mineral exploration, minesite reclamation and environmental issues. He is a graduate of McGill University, and is an AutoCAD Civil 3D Implementation Certified Expert. You can reach him at jmorrow@rand.com.

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